FINDING A FEASIBLE FLOW IN A NETWORK

Let G = (N, A) be a capacitated flow network, with b(i) the supply/demand at node i; we assume $\sum b(i) = 0$.

FEASIBLE FLOW PROBLEM:

Find flow **x**:

$$\sum_j x_{ij} - \sum_j x_{ji} = b(i) \quad \text{for each } i \in N$$

$$0 \le x_{ij} \le u_{ij} \quad \text{for all } (i,j) \in A$$

1. Construct the *transformed network* G′:

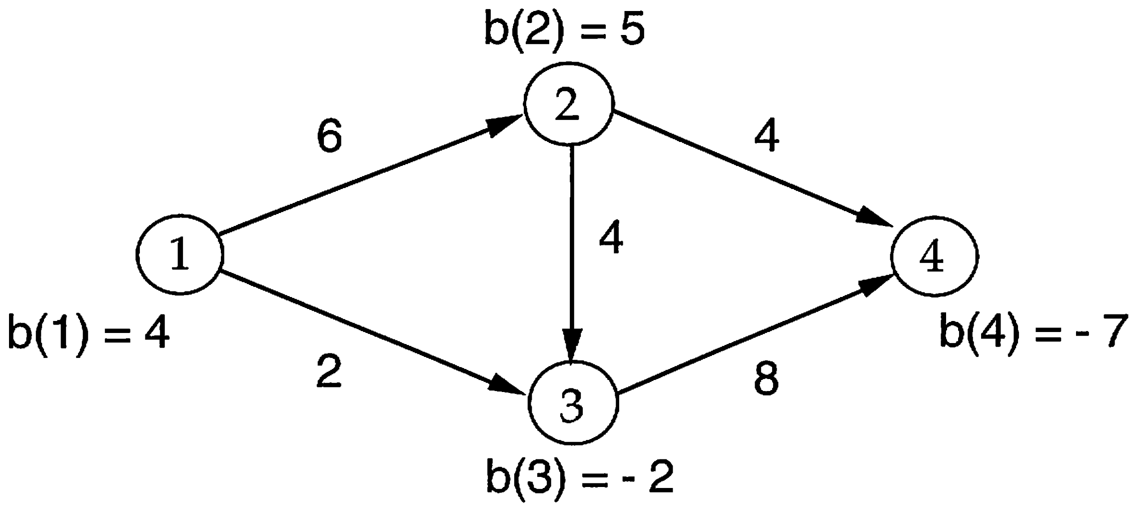
Introduce a source node s, and a sink node t.

> if b(i) > 0, add the arc (s, i) with capacity b(i) > 0;
> if b(i) < 0, add the arc (i, t) with capacity −b(i) > 0.

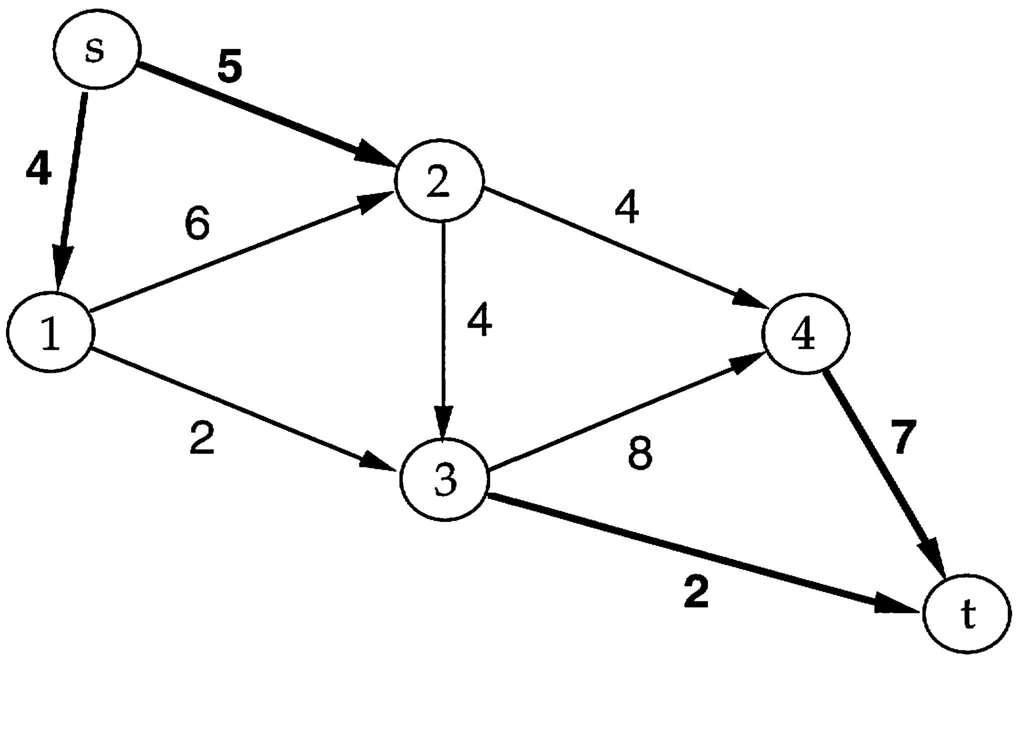
2. Then solve the maximum flow problem from s to t in the transformed network G′.

RESULT. If the maximum flow in G′ *saturates* all the source and sink arcs in G′ then the original problem has a feasible solution; otherwise it is infeasible.

EXAMPLE 1

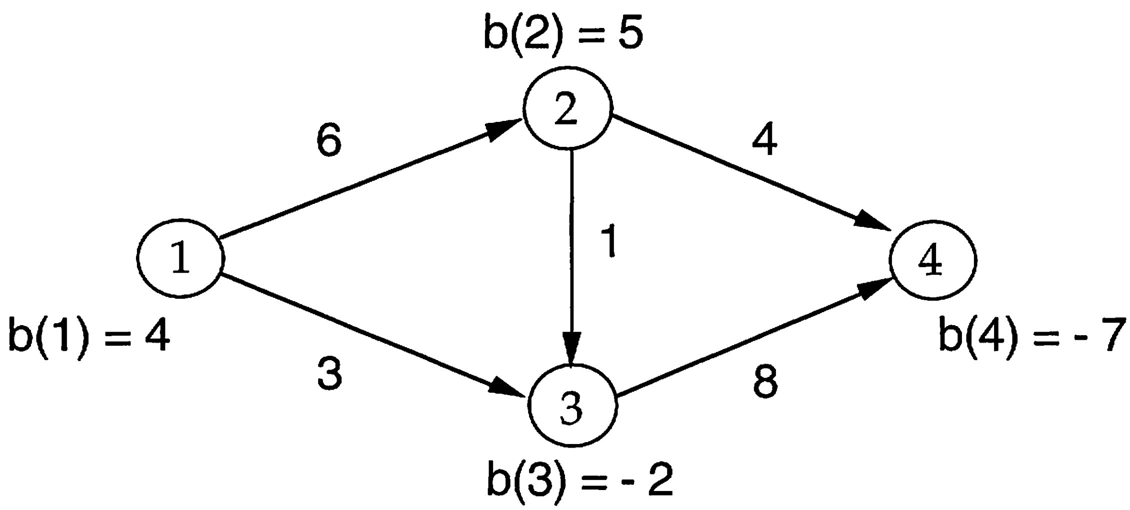


Find a maximum flow in G´:

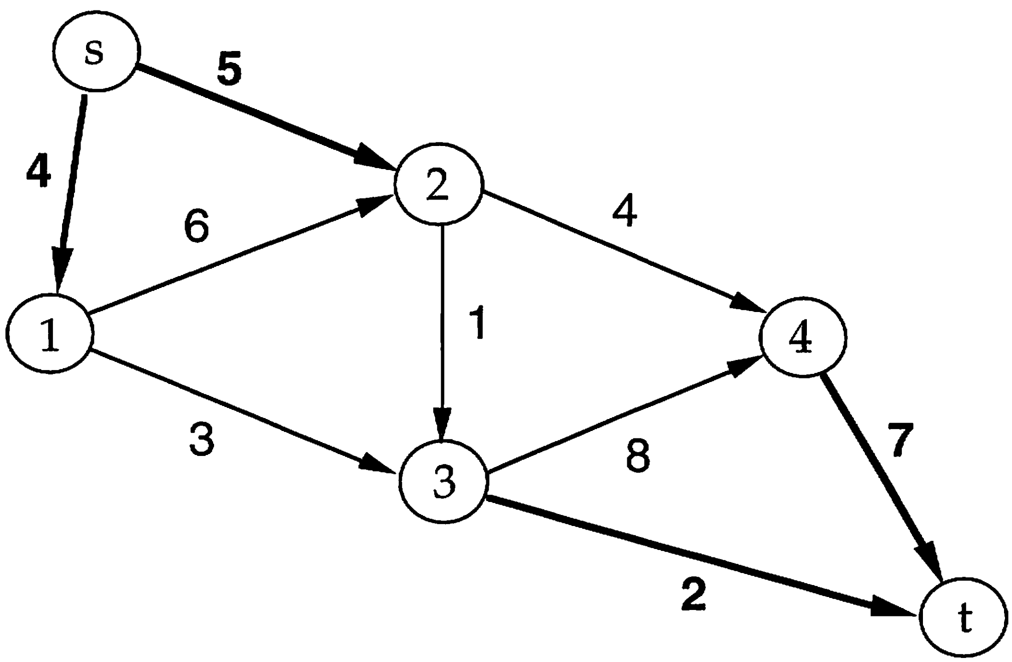


The original flow problem has a feasible solution.

EXAMPLE 2



Find a maximum flow in G´:



The original flow problem has no feasible solution.